

☺ 3.6 – Linear Systems ☺

Objectives:

1. Examine problems involving two or more conditions that must be satisfied at the same time.
2. Understand the visual representation of a solution to a system of equations and solve systems of equations graphically.
3. Solve systems of equations using the substitution property of equality.
4. Solve systems of equations using a table.
5. Estimate solutions from a graph.

System of Equations: A set of two or more equations that share the same variables and are solve or studied simultaneously.

Ways to solve a system of equations:

1. Graphing - Find the point of intersection
2. Substitution
3. Elimination

Example 1: Minh and Daniel are starting a business together, and they need to decide between long-distance phone carriers. One company offers the Phrequent Phoner Plan, which costs 20¢ for the first minute of a phone call and 17¢ for each minute after that. A competing company offers the Small Business Plan, which costs 50¢ for the first minute and 11¢ for each additional minute. Which plan should Minh and Daniel choose?

a. Write equations for each plan:

$x = \text{number of minutes (ind)}$
 $y = \text{total cost of call (dep)}$

$$\text{PPP} \rightarrow y = 20 + 17x$$

$$\text{SBP} \rightarrow y = 50 + 11x$$

b. Solve by graphing. What does the solution mean?

$(5, 105)$ It is the length of the phone call (5 minutes) when both plans cost the same (\$1.05).

c. Solve symbolically.

$$\begin{aligned} y &= 20 + 17x \\ y &= 50 + 11x \end{aligned}$$

$$\begin{aligned} 20 + 17x &= 50 + 11x \\ -11x & \quad -11x \end{aligned}$$

$$\begin{aligned} 20 + 6x &= 50 \\ -20 & \quad -20 \\ \hline 6x &= 30 \\ \hline x &= 5 \end{aligned}$$

$$\begin{aligned} y &= 20 + 17(5) \\ &= 20 + 85 \\ y &= 105 \end{aligned}$$

Example 2: Solve the following system of equations symbolically (algebraically). Then check your answer by graphing.

$$y = -2x + 1$$

$$y = x + 10$$

$$\begin{aligned} -2x + 1 &= x + 10 \\ +2x \quad +2x & \\ 1 &= 3x + 10 \\ -9 &= 3x \\ -3 &= x \end{aligned}$$

$$\begin{aligned} y &= -3 + 10 \\ y &= 7 \\ (-3, 7) \end{aligned}$$

Review Parallel and Perpendicular Lines:

Looking at the equations of two lines, how can we tell if they are parallel?

SAME SLOPE

Parallel lines are lines in the same plane that never intersect. They are always the same distance apart. You draw arrowheads on the middle of each line to show that they are parallel. You may have noticed a relationship between the slopes of parallel lines earlier in this course.



Looking at the equations of two lines, how can we tell if they are perpendicular?

*SLOPES ARE OPPOSITE
RECIPROCAL OF EACH OTHER*

Perpendicular lines are lines that meet at a right angle, that is, at an angle that measures 90° . In fact, four right angles are formed where perpendicular lines intersect. You draw a small box in one of the angles to show that the lines are perpendicular.



Example 3: Determine if the lines below are parallel, perpendicular or neither. Explain your answer:

a. $y = 5 - 2x$
 $y = -4 + 2x$

$m = -2$ $m = 2$

NEITHER

b. $y = \frac{2}{3}x + 6$
 $3x + 2y = -4$

$m = \frac{2}{3}$

$m = -\frac{3}{2}$

PERPENDICULAR

$3x + 2y = -4$

$2y = -3x - 4$

$y = -\frac{3}{2}x - 2$

c. $3y = 9x + 12$
 $-5y = -15x - 20$

$y = 3x + 4$

$3y = 9x + 12$

$y = 3x + 4$

NEITHER - SAME SLOPE AND SAME INTERCEPT
SAME LINE!

$m = 3$

$m = 3$

Example 4: Write an equation for each line described.

a. Perpendicular to $y = -3 + 2x$ and passing through the point $(5, -4)$.

$m = -\frac{1}{2}$ $y = -4 - \frac{1}{2}(x - 5)$

b. Parallel to $y = 7 - 5x$ and passing through the point $(-3, 9)$.

$m = -5$ $y = 9 - 5(x - (-3))$
 $y = 9 - 5(x + 3)$